Př: $\lim\limits_{n\to\infty} \dfrac{\overset{A_n}{\sqrt[3]{8^n-n^2}} - \overset{B_n}{\sqrt[5]{32^n-3^n}}}{\underset{C_n}{\sqrt{4^n+n}} - \underset{D_n}{\sqrt[4]{16^n+2.4^n n}}}\, 2^{-n} = \lim\limits_{n\to\infty} \dfrac{A_n^{15}-B_n^{15}}{C_n^4-D_n^4}\cdot\dfrac{C_n^3+\dots+D_n^3}{A_n^{14}+\dots+B_n^{14}}\,2^{-n} =$

$$\lim_{n\to\infty} \frac{\cancel{n^2}\,2^{12n}\,\overbrace{\frac{A_n^{15}-B_n^{15}}{n^2 2^{12n}}}^{\to -5\ (+)}}{\cancel{n^2}} \cdot \frac{2^{3n}\Big(\overbrace{\big(\tfrac{C_n}{2^n}\big)^3+\dots+\big(\tfrac{D_n}{2^n}\big)^3}^{\to 1}\Big)}{2^{14n}\Big(\big(\tfrac{A_n}{2^n}\big)^{14}+\dots+\big(\tfrac{B_n}{2^n}\big)^{14}\Big)}\,\cancel{2^{-n}} \overset{(+),(*),AL}{=} \frac{-5}{1}\cdot\frac{4}{15} = \underline{-\frac{4}{3}}$$

$$A_n^{15}-B_n^{15} = (8^n-n^2)^5 - (32^n-3^n)^3 = 2^{15n} - 5n^2 2^{12n} + \dots - (2^{15n} - 3.3^n.2^{10n} + \dots) = 3^{n+1}.2^{10n}$$

$$-5n^2.4^n.2^{10n} + P(n) \Rightarrow \frac{A_n^{15}-B_n^{15}}{n^2.2^{12n}} \to -5 \quad (+)$$

$$C_n^4 - D_n^4 = 16^n + 2n4^n + n^2 - 16^n - 2n4^n = n^2$$

$$\lim_{n\to\infty}\frac{A_n}{2^n} \overset{(ZOL)}{=} \sqrt[3]{\lim_{n\to\infty}\frac{8^n-n^2}{8^n}} = 1;\ \lim_{n\to\infty}\frac{B_n}{2^n} \overset{(ZOL)}{=} \sqrt[5]{\lim_{n\to\infty}\frac{32^n-3^n}{32^n}} = 1$$

$(*)$

$$\lim_{n\to\infty}\frac{C_n}{2^n} \overset{ZOL}{=} \sqrt{\lim_{n\to\infty}\frac{4^n+n}{4^n}} = 1;\ \lim_{n\to\infty}\frac{D_n}{2^n} = \sqrt[4]{\lim_{n\to\infty}\frac{16^n+2.4^n n}{16^n}} = 1$$

Př: $\lim\limits_{n\to\infty}\Big(\overset{A_n}{\sqrt[3]{n^{48}+n}} - \overset{B_n}{\sqrt[3]{n^{48}+n^2}}\Big)\overbrace{\big((n^3+3)^{12} - (n^4+4n)^9\big)}^{C_n} =$

$$\lim_{n\to\infty}\frac{n-n^2}{A_n^2+A_nB_n+B_n^2}\cdot C_n = \lim_{n\to\infty}\frac{\cancel{n^2}\big(\frac{1}{n}-1\big)}{\cancel{n^{32}}\Big(\big(\frac{A_n}{n^{16}}\big)^2+\dots+\big(\frac{B_n}{n^{16}}\big)^2\Big)}\cdot\frac{C_n}{n^{30}}\cdot\cancel{n^{30}} \overset{AL,(+),(*)}{=} \frac{0-1}{1+1+1}\cdot 18$$

$$= \underline{\underline{-6}}$$

$$C_n = \cancel{n^{36}} + \cancel{3.12\,n^{33}} + 9.\binom{12}{2}n^{30} + P(n) - \big(\cancel{n^{36}} + \cancel{4.9.\,n^{33}} + (4n)^2\binom{9}{2}n^{28} + Q(n)\big)$$

$$= n^{30}.\big(\underbrace{6.11.9 - 9.4.4^2}_{9(66-64)=18}\big) + P(n) - Q(n) \Rightarrow \lim_{n\to\infty}\frac{C_n}{n^{30}} = 18\ (+)$$

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$$\lim_{n\to\infty}\frac{A_n}{n^{16}} \overset{(ZOL)}{=} \cancel{\lim_{n\to\infty}\sqrt[3]{\ }}\ \sqrt[3]{\lim_{n\to\infty} 1+\frac{1}{n^{47}}} = 1,\ \lim_{n\to\infty}\frac{B_n}{n^{16}} \overset{(ZOL)}{=}$$

$$\sqrt[3]{\lim_{n\to\infty} 1+\frac{1}{n^{46}}} = 1 \qquad (*)$$